MATH

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PURPOSE OF THIS GUIDE

This "Study Guide" is designed as a basic review of the math principles that are commonly used in electrical and mechanical studies.

BASIC MATH RULES

ADDITION AND SUBTRACTION

A. To add two numbers of the same sign, add their absolute values and attach the common sign.
B. To add two numbers of opposite signs, subtract the smaller absolute value from the larger absolute value and attach the sign of the larger.
C. To subtract signed numbers, change the sign of the number to be subtracted (subtrahend) and add as in (A) or (B) above.

MULTIPLICATION AND DIVISION

A. To multiply two signed numbers, multiply their absolute values and attach a positive if they have like signs, a negative if they have unlike signs.
B. To divide two signed numbers, use rule (A) above, dividing instead of multiplying.

Examples:

A. Addition

1. $-9 + (-5) = -14$
2. $-9 + 5 = -4$
3. $3.65 + (-1.27) = 2.38$

B. Multiplication

1. $(-9) \times (-5) = 45$
2. $(-9) \times 5 = -45$
3. $2.80 \times (-1.25) = -3.5$

C. Division

1. $(-24) \div 3 = -8$
2. $(-9) \div (-3/2) = 6$
3. $1.6 \div (-0.4) = -4$

COMMON FRACTIONS

BASIC CONCEPTS

FRACTIONS AND MEASUREMENT

The need for greater precision in measurement led to the concept of fractions. For example, "the thickness is $^3/_4$ in." is a more precise statement than "the thickness is between 0 and 1 in." In the first measurement, the space between the inch marks on the scale was likely subdivided into quarters; on the second scale, there were no subdivisions. In the metric system, the subdivisions are multiples of 10 and all measurements are expressed as *decimal* fractions. In the British system, the subdivisions are *not* multiples of 10 and the measurements are usually recorded as common fractions. The universal use of the metric system would greatly simplify the making and recording of measurements. However, common fractions would still be necessary for algebraic operations.

TERMS

Fraction: Numbers of the form 3/4, 1/2, 6/5 are called fractions. The line separating the two integers indicates division.

Numerator: (or dividend) is the integer above the fraction line.

Denominator: (or divisor) is the integer below the fraction line. The denominator in a measurement may show the number of subdivisions in a unit.

Common fraction: a fraction whose denominator is numbers other than 10, 100, 1000, etc. Other names for it are: simple fraction and vulgar fraction.
Examples:

$$\frac{1}{125}, \frac{2}{7}, \frac{15}{32}$$

Decimal fraction: a fraction whose denominator has some power of 10.

Mixed number: is a combination of an integer and a fraction.
Example:

$$\text{The mixed number } 3\frac{2}{5} \text{ indicates the addition of } 3 + \frac{2}{5}$$

Improper Fraction: Not in lowest terms **Example:** $\frac{9}{2}$

BASIC PRINCIPLE

A fundamental principle used in work with fractions is: *If both the numerator and denominator of a fraction are multiplied or divided by the same non-zero number, the value of the fraction is unchanged.* Another way of expressing this rule is: If a fraction is multiplied by 1, the value of the fraction remains unchanged.

Example:

$$\frac{2}{3} = \frac{2 \bullet 2}{3 \bullet 2} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{2 \bullet 5}{3 \bullet 5} = \frac{10}{15}$$

$$\frac{2}{3} = \frac{2 \bullet 6}{3 \bullet 6} = \frac{12}{18}$$

The numerator and denominator of the fraction 2/3 were multiplied by 2, 5, and 6 respectively. The fractions 2/3, 4/6, 10/15, 12/18 are equal and are said to be in *equivalent forms*. It can be seen that in the above example the change of a fraction to an equivalent for implies that the fraction was multiplied by 1. Thus the multipliers 2/2, 5/5, 6/6 of the fraction 2/3 in this case are each equal to 1.

REDUCTION TO LOWEST TERMS

The basic principle given on the previous page allows us to simplify fractions by dividing out any factors which the numerator and denominator of a fraction may have in common. When this has been done, the fraction is in reduced form, or reduced to its lowest terms. This is a simpler and more convenient form for fractional answers. The process of reduction is also called cancellation.

Example:

Since $\frac{18}{30} = \frac{2 \bullet 3 \bullet 3}{2 \bullet 3 \bullet 5}$*, the numerator and demoninator*

can be divided by the common factors 2 and 3.

The resulting fraction is $\frac{3}{5}$*, which is the reduced*

form of the fraction $\frac{18}{30}$*.*

Factoring

The process of factoring is very useful in operations involving fractions. If an integer greater than 1 is not divisible by any positive integer except itself and 1, the number is said to be prime. Thus, 2, 3, 5, 7, etc., are prime numbers, the number 2 being the only even prime number.

If a number is expressed as the product of certain of its divisors, these divisors are known as factors of the representation. The prime factors of small numbers are easily found by inspection. Thus, the prime factors of 30 are 2, 3, 5. The following example illustrates a system which can be used to find the prime factors of a large number.

Example: Find the prime factors of 1386.

Try to divide 1386 by each of the small prime numbers, beginning with 2. Thus, 1386 / 2 = 693. Since 693 is not divisible by 2, try 3 as a divisor: 693 ¸ 3 = 231. Try 3 again: 231 ¸ 3 = 77. Try 3 again; it is not a divisor of 77, and neither is 5. However, 7 is a divisor, for 77 ¸ 7 = 11. The factorization is complete since 11 is a prime number. Thus, $1386 = 2 \cdot 3 \cdot 3 \cdot 7 \cdot 11$. The results of the successive divisions might be kept in a compact table shown below.

Dividends	1386	693	231	77	11
Factors	2	3	3	7	11

The following divisibility rules simplify factoring:

Rule 1. A number is divisible by 2 if its last digit is even.
Example: The numbers 64, 132, 390 are each exactly divisible by 2.

Rule 2. A number is divisible by 3 if the sum of its digits is divisible by 3.
Example: Consider the numbers 270, 321, 498. The sums 9, 6, 21 of the digits are divisible by 3.

Rule 3. A number is divisible by 5 if its last digit is 5 or zero.
Example: The numbers 75, 135, 980 are each divisible by 5.

Rule 4. A number is divisible by 9 if the sum of its digits is divisible by 9.
Example: The numbers 432, 1386, and 4977 are exactly divisible by 9 since the sums of their digits are 9, 18, 27, and these are divisible by 9.

OPERATIONS WITH FRACTIONS

ADDITION OF FRACTIONS

To add fractions, which have a common denominator, add their numerators and keep the same common denominator.

Example: *Determine the sum of:* $\frac{1}{4} + \frac{5}{4} + \frac{3}{4}$.

Adding the numerators, $1 + 5 + 3 = 9$.

Keeping the same common denominator 4,

the desired result is $\frac{9}{4}$ *or* $2\frac{1}{4}$.

To add fractions with unlike denominators, determine the least common denominator. Express each fraction in equivalent form with the **LCD**. Then perform the addition.

Example: *Determine the sum of:* $\frac{1}{2} + \frac{3}{4} + \frac{2}{8}$. *The LCD is 8.*

The sum with the fractions in equivalent form is $\frac{4}{8} + \frac{6}{8} + \frac{2}{8}$.

Adding the numerators and keeping the same LCD,

the result is $\frac{12}{8}$ *or* $\frac{3}{2}$ *or* $1\frac{1}{2}$.

To add mixed numbers, calculate the sum of the integers separately from the sum of the fractions. Then add the two sums.

Example: *Add the mixed numbers* $7\frac{1}{6}$ *and* $2\frac{3}{4}$.

First we add the integers: $7 + 2 = 9$.

Adding the fractions, we get $\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$.

Combining the two sums, the answer we obtain is $9\frac{11}{12}$.

SUBTRACTION OF FRACTIONS

As with addition, before two fractions can be subtracted they must have a common denominator.

Example: *Subtract* $\frac{2}{3}$ *from* $\frac{3}{4}$. *The LCD is 12* .

The subtraction with the fractions in equivalent

form gives: $\frac{3 \cdot 3}{4 \cdot 3} - \frac{2 \cdot 4}{3 \cdot 4} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Example: *Perform the operation* $7\frac{1}{5} - 4\frac{5}{10}$.

The LCD of the fractions is 10 .

The desired difference can be expressed as $7\frac{2}{10} - 4\frac{5}{10}$.

But $\frac{5}{10}$ *is larger than* $\frac{2}{10}$. *Therefore, borrowing 1 or* $\frac{10}{10}$

from 7 and adding it to $\frac{2}{10}$, *and performing the operation,*

we obtain: $6\frac{12}{10} - 4\frac{5}{10} = 2\frac{7}{10}$.

MULTIPLICATION OF FRACTIONS

To multiply two or more fractions, multiply their numerators to obtain the numerator of the product. Multiply the denominators to obtain the denominator of the product.

Example: *Multiply* $\frac{3}{7} \text{ x } \frac{2}{5}$.

The desired product is $\frac{3 \text{ x } 2}{7 \text{ x } 5} = \frac{6}{35}$.

The calculations are greatly simplified, and the product put in its lowest terms, by cancellation of common factors.

Example: *Determine the product* $\frac{2}{5} \text{ x } \frac{10}{3} \text{ x } \frac{6}{7}$.

Express all the numerators and denominators in factored form,

cancel the common factors, and carry out the multification

of the remaining factors . The result is:

$$\frac{2}{\dot{5}} \text{ x } \frac{2 \cdot \dot{5}}{\dot{3}} \text{ x } \frac{2 \cdot \dot{3}}{7} = \frac{8}{7} = 1\frac{1}{7}.$$

Note: The $\dot{x}$ is used to show the numbers being cancelled.

Every integer can be written as a fraction with 1 as a denominator, since an integer divided by 1 is equal to the integer. Thus, to multiply an integer by a fraction, multiply the numerator of the fraction by the integer and retain the denominator of the given fraction.

Example: *Determine* $\frac{3}{5} \text{ x } 6$.

Applying the rule, $\frac{3}{5} \text{ x } \frac{6}{1} = \frac{3 \text{ x } 6}{5 \text{ x } 1} = \frac{18}{5} = 3\frac{3}{5}$.

To multiply mixed numbers, convert them first into fractions and then apply the rule for multiplication of fractions.

Example: *Multiply* $1\frac{3}{4}$ *by* $3\frac{1}{7}$.

Converting $1\frac{3}{4}$ *and* $3\frac{1}{7}$ *to common fractions,*

we get $1\frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$ *and* $3\frac{1}{7} = \frac{21}{7} + \frac{1}{7} = \frac{22}{7}$.

Now in fractional form $\frac{7}{4} \times \frac{22}{7}$

Express in factored form $\frac{\dot{7}}{2 \cdot \dot{2}} \times \frac{\dot{2} \cdot 11}{\dot{7}}$. *Cancel out 7 and 2.*

The result is $\frac{11}{2}$ *or* $5\frac{1}{2}$.

DIVISION OF FRACTIONS

To divide one fraction by another, invert the divisor and multiply the fractions.

Example: $3/5 \div 4/7 \Rightarrow$ *invert the divisor* $4/7$ *to* $7/4 \Rightarrow$

Now multiply $3/5 \times 7/4 = \frac{3 \times 7}{5 \times 2 \times 2} = \frac{21}{20} \Rightarrow$

Notice nothing cancelled and the result is $\frac{21}{20}$

To divide mixed numbers, convert them first to fractions as before and apply the rule for division.

DIVISION BY ZERO

It is important to emphasize the following reminder: **Division by 0 is impossible.** Therefore, a fraction cannot have 0 as a denominator.

RECIPROCALS

The reciprocal of a number is 1 divided by the number. It follows from the definition that the product of a number and its reciprocal is 1. The reciprocal of zero is not defined.

Example: *The reciprocal of 25 is* $\frac{1}{25}$ *or 0.04.*

Conversely, the reciprocal, of 0.04 is $\frac{1}{0.04}$ *or 25.*

CONVERSION OF COMMON FRACTIONS INTO DECIMAL FRACTIONS

To convert a common fraction into a decimal equivalent form, divide the numerator by the denominator.

Example: *Convert* $\frac{7}{16}$ *into a decimal fraction.*

Since 16 is larger than 7, there will be no digits except 0 to the left of the decimal point in the quotient. Then performing the division, 0 is adjoined to 7 to give 70, which is larger than 16.

Proceeding as with integers, the decimal equivalent of $\frac{7}{16}$ *is 0.4375.*

PRACTICE PROBLEMS

1. Solve:

$$\frac{9}{64} - \frac{1}{16} =$$

2. Solve:

$$12\frac{7}{8} \div 4\frac{2}{3} =$$

3. Solve:

$$1\frac{3}{4} + 5\frac{1}{6} =$$

4. Solve:

$$\frac{4}{9} x \frac{8}{12} =$$

5. Solve:

$$\frac{8}{4} + \frac{3}{32} =$$

6. Solve:

$$0.5 \div \frac{1}{4} =$$

FUNDAMENTAL RULES OF ALGEBRA

Commutative Law of Addition	$a + b = b + a$
	$9 + 1 = 1 + 9$
Associative Law of Addition	$a + (b + c) = (a + b) + c$
	$4 + (3 + 2) = (4 + 3) + 2$
Commutative Law of Multiplication	$ab = ba$
	$6 \times 7 = 7 \times 6$
Associative Law of Multiplication	$a\,(bc) = (ab)\,c$
	$4(5 \times 2) = (4 \times 5)2$
Distributive Law	$a\,(b + c) = ab + ac$
	$2\,(3 + 9) = 2 \times 3 + 2 \times 9$

SOLVING EQUATIONS AND FORMULAS

An arithmetic equation such as *3 + 2 = 5* means that the number named on the left (3 + 2) is the same as the number named on the right (5).

An algebraic equation, such as $x + 3 = 7$, is a statement that the sum of some number x and *3* is equal to *7*. If we choose the correct value for x, then the number $x + 3$ will be equal to *7*. x is a *variable*, a symbol that stands for a number in an equation, a blank space to be filled. Many numbers might be put in the space, but only one makes the equation a true statement.

Find the missing numbers in the following arithmetic equations:

a)	37 + _______ = 58	b)	_______ - 15 = 29
c)	4 x _______ = 52	d)	28 ÷ _______ = 4

We could have written these equations as

a)	37 + A = 58	b)	B - 15 = 29
c)	4 x C = 52	d)	28 ÷ D = 4

Of course any letters would do in place of A, B, C, and D in these algebraic equations.

How did you solve these equations? You probably eye-balled them and then mentally juggled the other information in the equation until you had found a number that made the equation true. Solving algebraic equations is very similar except that we can't "eyeball" it entirely. We need certain and systematic ways of solving the equation that will produce the correct answer quickly every time.

Each value of the *variable* that makes an equation true is called a *solution* of the equation. For example, the solution of $x + 3 = 7$ is $x = 4$.

Check: **(4) + 3 = 7**

Another **example**: The solution of ***2x - 9 = 18 - 7x is x = 3***

Check:

$$\begin{aligned} 2(3) - 9 &= 18 - 7(3) \\ 6 - 9 &= 18 - 21 \\ -3 &= -3 \end{aligned}$$

For certain equations more than one value of the variable may make the equation true. For example, the equation $x^2 + 6 = 5x$ is true for $x = 2$.

Check: $(2)^2 + 6 = 5(2)$
$4 + 6 = 5 \times 2$
$10 = 10$

It is also true for $x = 3$,

Check: $(3)^2 + 6 = 5(3)$
$9 + 6 = 5 \times 3$
$15 = 15$

Use your knowledge of arithmetic to find the solution of each of the following:

a) $4 + x = 11$ b) $x - 1 = 6$

c) $x + 2 = 9$ d) $8 - x = 1$

Solving Equations

Equations as simple as the ones above are easy to solve by guessing, but guessing is not a very dependable way to do mathematics. We need some sort of rule that will enable us to rewrite the equation to be solved. The general rule is to treat every equation as a balance of the two sides. Any changes made in the equation must not disturb this balance. **Any operation performed on one side of the equation must also be performed on the other side.**

Two kinds of balancing operations may be used.

1. Adding or subtracting a number on both sides of the equation does not change the balance.

2. Multiplying or dividing both sides of the equation by a number (**but not zero**) does not change the balance.

Example: $x - 4 = 2$

We want to change this equation to an equivalent equation with only x on the left, so we add 4 to each side of the equation.

$x - 4 + 4 = 2 + 4$ Next we want to combine like terms.

$x - 0 = 6$

$x = 6$ Solution.

Check: $x - 4 = 2$

$6 - 4 = 2$

$2 = 2$

Use these balancing operations to solve the equations below:

a) $8 + x = 14$ b) $x - 4 = 10$

c) $12 + x = 27$ d) $x + 6 = 2$

Solving many simple algebraic equations involves both kinds of operations: addition/subtraction and multiplication/division. **For example**,

Solve:	$2x + 6 = 14$	We want to change this equation to an equivalent equation with only x or terms that include x on the left, so subtract 6 from both sides.
	$2x + 6 - 6 = 14 - 6$	Combine like terms.
	$2x = 8$	Now change this to an equivalent equation with only x on the left by dividing both sides by 2.
	$2x \div 2 = 8 \div 2$	$2x \div 2 = x$ and $8 \div 2 = 4$
	$x = 4$	

Use these balancing operations to solve the equations below:

a) $3x - 7 = 11$

b) $7x + 2 = 51$

c) $x - 6 = 3x$

d) $12 - 2x = 4 - x$

e) $17x = 0$

f) $3 + 2x = 17$

g) $5y + 3 = 28$

h) $(x - 2) - (4 - 2x) = 12$

Transposing Equations

To transpose an equation or formula so as to solve for a different unknown means to rewrite the formula as an equivalent formula with that letter isolated on the left of the equals sign.

For example, the area of the triangle is given by the formula

$$A = \frac{BH}{2}$$

where A is the area, B is the length of the base, and H is the height.

Solving for the base B gives the equivalent formula

$$B = \frac{2A}{H}$$

Solving for the height H gives the equivalent formula

$$H = \frac{2A}{B}$$

Solving formulas is a very important practical application of algebra. Very often a formula is not written in the form that is most useful. To use it you may need to rewrite the formula, solving it for the letter whose value you need to calculate.

To solve a formula, use the same balancing operations that you used to solve equations. You may add or subtract the same quantity on both sides of the formula and you may multiply or divide both sides of the formula by the same non-zero quantity.

For example, to solve the formula

$$S = \frac{R+P}{2} \quad for\ R$$

First, multiply both sides of the equation by **2**.

$$2 \bullet S = 2 \bullet (\frac{R+P}{2})$$

The 2's on the right cancel.

$$2S = R + P$$

Second, subtract **P** from both sides of the equation.

$$2S - P = R + P - P$$

The P's on the right cancel.

$$2S - P = R$$

This formula can be reversed to read.

$$R = 2S - P$$

We have solved the formula for **R**.

Remember, when using the multiplication/division rule, you must multiply or divide *all* of both sides of the formula by the same quantity. Practice solving formulas with the following problems:

a) $V = \dfrac{3K}{T}$ for K

b) $Q = 1 - R + T$ for R

c) $V = \pi R^2 H - AB$ for H

d) $P = \dfrac{T}{A - B}$ for A

e) $P = 2A + 3B$ for A

f) $E = MC^2$ for M

g)

$$S = \frac{A - RT}{1 - R} \quad for\ A$$

h)

$$S = \frac{1}{2}gt^2 \quad for\ g$$

i)

$$P = I^2 R \quad for\ R$$

j)

$$I = \frac{V}{R + a} \quad for\ R$$

k)

$$A = \frac{2V - W}{R} \quad for\ V$$

l)

$$F = \frac{9C}{5} + 32 \quad for\ C$$

m)

$$A = \frac{\pi R^2 S}{360} \quad for\ S$$

n)

$$P = \frac{t^2 dN}{3.78} \quad for\ d$$

o)

$$C = \frac{AD}{A + 12} \quad for\ D$$

p)

$$V = \frac{\pi LT^2}{6} + 2 \quad for\ L$$

Factoring and Adding/Removing Parentheses

Parentheses are used to group or isolate terms in an expression for special consideration. When two or more terms are enclosed in parentheses they are to be treated as a single quantity. A *term* is an expression in a quantity which is isolated by plus or minus signs.

Removal of Parentheses - Parentheses preceded by a plus sign can be removed without changing any signs. Parentheses preceded by a minus sign can be removed only if all the enclosed signs are changed.

Example: **A + (B + C - D) is the same as A + B + C - D**
A - (B + C - D) is the same as A - B - C + D

Parentheses Preceded by a Factor - These can be handled two ways.

a) By performing all operations within the parentheses first and then multiplying.
Example: A(4 + 5)
A(9)
9A

b) Or multiplying each term within the parentheses by the factor.
Example: A(4 + 5)
4A + 5A
9A The result is the same as above.

Factoring - In the expression **A(B + C - D)**, each term with the parentheses must be multiplied by the factor **A** with the result being
A(B + C - D)
AB + AC - AD

Conversely, if a series of terms have a common quantity, that quantity can be factored out.

Example: **AB + AC - AD** the factor **A** is common

A(B + C - D) result of factoring out **A**

Example: **4AB + 4AX - 4AY** factors **4** and **A** are common

4A(B + X - Y) result of factoring out **4A**

Using Square Roots in Solving Equations

The equations you have learned to solve so far are all *linear* equations. The variable appears only to the first power with no x^2 or x^3 terms appearing in the equations. Terms that are raised to a power greater than one can also be solved using the balancing operations from the previous lessons. As an example, let's solve the following equation.

$$c^2 = a^2 + b^2 \quad \text{for } c$$

First, we have to take the square root of both sides.

$$\sqrt{c^2} = \sqrt{a^2 + b^2}$$

The square root of C^2 is **C**, and the result is

$$c = \sqrt{a^2 + b^2}$$

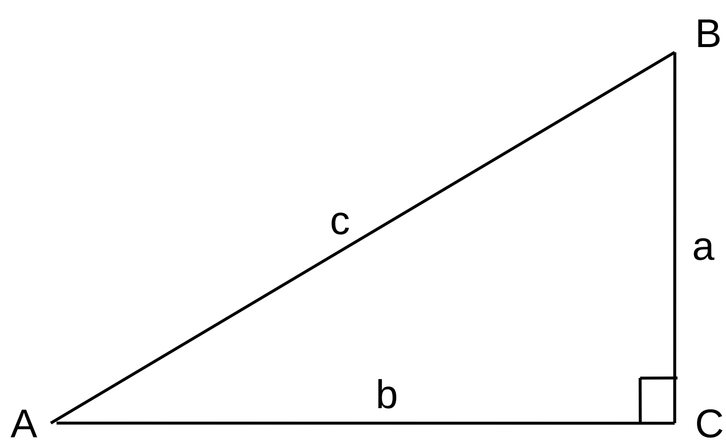
TRIANGLE TRIGONOMETRY

The word *trigonometry* means simply "triangle measurement". We know that the ancient Egyptian engineers and architects of 4000 years ago used practical trigonometry in building the pyramids. By 140 B.C. the Greek mathematician Hipparchus had made trigonometry a part of formal mathematics and taught it as astronomy. We will look at only the simple practical trigonometry used in electrical applications.

A *triangle* is a polygon having three sides. In this course, the electrical applications will only need to refer to a *right triangle*. A right triangle contains a 90° angle. The sum of the remaining two angles is 90°. Two of the sides are perpendicular to each other. The longest side of a right triangle is always the side opposite to the right angle. This side is called the *hypotenuse* of the triangle.

Labeling of Sides and Angles

The following figure indicates the conventional method for designating sides and angles:



Side **a** is the altitude of the triangle.
Side **b** is the base of the triangle.
Side **c** is the hypotenuse of the triangle.

* Note that each angle has the same letter designation as the opposite side. When the angle is unknown Greek letters are used to represent these angles. A common Greek letter used for angles is θ **(theta)**.

Pythagorean Theorem

The Pythagorean theorem is a rule or formula that allows us to calculate the length of one side of a right triangle when we are given the lengths of the other two sides. Although the formula is named after the ancient Greek mathematician Pythagoras, it was known to Babylonian engineers and surveyors more than a thousand years before Pythagoras lived.

Pythagorean Theorem - For any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

Trigonometric Functions

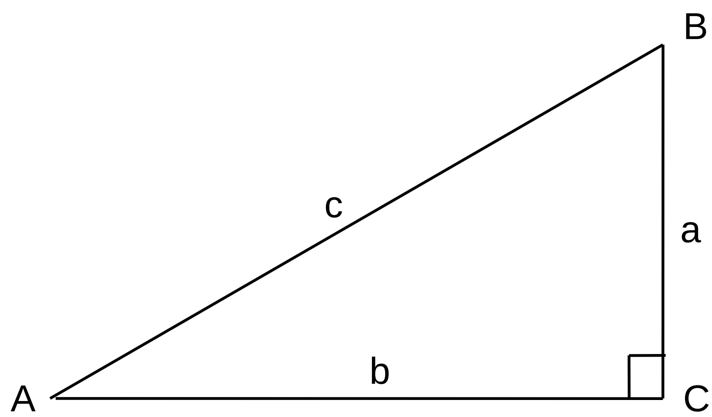
We have observed the relationship of the side lengths of a right triangle based upon the Pythagorean Theorem. There is also a relationship between the ratio of the opposite side to the hypotenuse for a fixed angle. The ratio will always be constant regardless the unit of measure of the sides. The same applies to the ratio of the adjacent side to the hypotenuse and the ratio of the opposite side to the adjacent side.

These ratios are useful in solving right triangles and certain other scientific calculations. These ratios are called *trigonometric functions*. Using the labeling techniques from the previous page, these functions are shown as follows:

$$sine \angle\theta = \frac{OppositeSide}{Hypotenuse}$$

$$cosine \angle\theta = \frac{Adjacent\ Side}{Hypotenuse}$$

$$tangent \angle\theta = \frac{OppositeSide}{Adjacent\ Side}$$



The sine of angle A is written:

$$sine\ \angle A = \frac{O}{H} = \frac{a}{c}$$

The cosine of angle A is written:

$$cosine\ \angle A = \frac{A}{H} = \frac{b}{c}$$

The tangent of angle A is written:

$$tangent\ \angle A = \frac{O}{A} = \frac{a}{b}$$

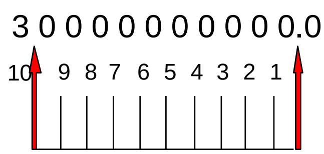
SCIENTIFIC NOTATION

In technical and scientific work, numbers are often encountered which are either very large or very small in magnitude. Illustrations of such numbers are given in the following examples.

Television signals travel at about 30,000,000,000 cm/second
The mass of the earth is about 6,600,000,000,000,000,000,000 tons
A typical coating used on aluminum is about 0.0005 inches thick
The wave length of some x-rays is about 0.000000095 cm

Writing numbers such as these is inconvenient in ordinary notation, as shown above, particularly when the numbers of zeros needed for the proper location of the decimal point are excessive. Therefore, a convenient notation, known as **scientific notation**, is normally used to represent such numbers. *A number written in scientific notation is expressed as the product of a number between 1 and 10 and a power of ten.* Basically, we can say that a number expressed in scientific notation will only have 1 digit left of the decimal point. Let's convert the numbers from above:

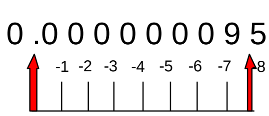
Television signals travel at about 30,000,000,000.0 cm/second
or 3 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 cm/second



or 3.0 x 10^{10} cm/second

Note: When moving the decimal point to the left, add one exponent for each place moved.

The wave length of some x-rays is about 0.000000095 cm
or 9.5 ¸ 10 ¸ 10 ¸ 10 ¸ 10 ¸ 10 ¸ 10 ¸ 10 ¸ 10 cm
or 9.5 x 10^{-8} cm



Note: When moving the decimal point to the right, subtract one from exponent for each place moved.

ADDITION AND SUBTRACTION USING SCIENTIFIC NOTATION

Before adding or subtracting two numbers expressed in scientific notation, their powers of 10 must be equal.

Example: **Add:** $1.63 \times 10^4 + 2.8 \times 10^3$

Convert to: $16.3 \times 10^3 + 2.8 \times 10^3 = 19.1 \times 10^3$

or: $1.63 \times 10^4 + .28 \times 10^4 = 1.91 \times 10^4$

MULTIPLICATION AND DIVISION USING SCIENTIFIC NOTATION

To multiply two numbers expressed in scientific notation, multiply their decimal numbers together first, then add their exponents. If the decimal number is greater than 9 after the multiplication, move the decimal point and compensate the exponent to express in scientific notation.

$$8.4 \text{ X } 10^3 \times 2.2 \text{ X } 10^2 = 18.48 \text{ X } 10^5 = 1.848 \times 10^6$$

To divide two numbers expressed in scientific notation, divide separately their decimal numbers and subtract their exponents.

$$8.4 \text{ X } 10^3 \div 2.0 \text{ X } 10^2 = 4.2 \text{ X } 10^1$$

POWERS OF TEN NOTATION USING ENGINEERING UNITS

Powers of ten notation follows the same rules for addition, subtraction, multiplication, and division as scientific notation. The only difference is that the resultant for powers of ten does not have to be expressed with only 1 digit to the left of the decimal point. When using engineering units for calculations, this is very important. Below is a table for engineering units expressed in powers of ten:

Power	Decimal Number	Prefix	Abbreviation
1×10^9	1,000,000,000	Giga	(G)
1×10^6	1,000,000	Mega	(M)
1×10^3	1,000	Kilo	(K)
1×10^0	1	Unity	
1×10^{-3} .001		Milli	(m)
1×10^{-6} .	000,001	Micro	(μ)
1×10^{-9} .	000,000,001	Nano	(n)
1×10^{-12}	.000,000,000,001	Pico	(p)

Example: $V = I \times R$

1.2 Kvolts = 200 mAmps × 6 KW

APPROXIMATE NUMBERS AND SIGNIFICANT DIGITS

When we perform calculations on numbers, we must consider the accuracy of these numbers, since this affects the accuracy of the results obtained. Most of the numbers involved in technical work are approximate, having been arrived at through some process of measurement. However, certain other numbers are exact, having been arrived at through some definition (1 hour = 60 minutes) or counting process (cured tires counted). We can determine whether or not a number is approximate or exact if we know how the number was determined.

Most numbers used in the calculations for this course will be approximate. In calculations using approximate numbers, the position of the decimal point as well as the number of significant digits is important. What are significant digits? Except for the left-most zeros, all digits are considered to be **significant digits.**

The last significant digit of an approximate number is known not to be completely accurate. It has usually been determined by estimation of **rounding off**. However, we do know that it is at most in error by one-half of a unit in its place value.

> ***The principle of rounding off a number is to write the closest approximation, with the last significant digit in a specified position, or with a specified number of significant digits.***

Let's specify this process of rounding off as follows: If we want a certain number of significant digits, we examine the digit in the next place to the right. If this digit is **less than 5**, we accept the digit in the last place. If the next digit is **5 or greater**, we increase the digit in the last place by 1, and this resulting digit becomes the final significant digit of the approximation.

The answers on the assessment are taken to the fourth digit and round back to three places right of the decimal. Please utilize this format during this class.